OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE UNIT **4725/01**

MATHEMATICS

Further Pure Mathematics 1

THURSDAY 18 JANUARY 2007 Afternoon

Time: 1 hour 30 minutes

Additional Materials: Answer Booklet (8 pages)
List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- **You are reminded of the need for clear presentation in your answers.**

This document consists of **4** printed pages.

[Turn over

1 The matrices **A** and **B** are given by $\mathbf{A} = \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} a & -1 \\ -3 & -2 \end{pmatrix}$.

(i) Given that $2\mathbf{A} + \mathbf{B} = \begin{pmatrix} 1 & 1 \\ 3 & 2 \end{pmatrix}$, write down the value of a. [1]

(ii) Given instead that $\mathbf{AB} = \begin{pmatrix} 7 & -4 \\ 9 & -7 \end{pmatrix}$, find the value of a. [2]

2 Use an algebraic method to find the square roots of the complex number $15 + 8\mathrm{i}$. [6]

3 Use the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^3$ to find

$$\sum_{r=1}^{n} r(r-1)(r+1),$$

expressing your answer in a fully factorised form. [6]

4 **(i)** Sketch, on an Argand diagram, the locus given by $|z - 1 + \mathrm{i}| = \sqrt{2}$. [3]

(ii) Shade on your diagram the region given by $1 \leqslant |z - 1 + \mathrm{i}| \leqslant \sqrt{2}$. [3]

5 **(i)** Verify that $z^3 - 8 = (z - 2)(z^2 + 2z + 4)$. [1]

(ii) Solve the quadratic equation $z^2 + 2z + 4 = 0$, giving your answers exactly in the form $x + \mathrm{i}y$. Show clearly how you obtain your answers. [3]

(iii) Show on an Argand diagram the roots of the cubic equation $z^3 - 8 = 0$. [3]

6 The sequence $u_1, u_2, u_3, \ldots$ is defined by $u_n = n^2 + 3n$, for all positive integers n.

(i) Show that $u_{n+1} - u_n = 2n + 4$. [3]

(ii) Hence prove by induction that each term of the sequence is divisible by 2. [5]

7 The quadratic equation $x^2 + 5x + 10 = 0$ has roots α and β.

(i) Write down the values of $\alpha + \beta$ and $\alpha\beta$. [2]

(ii) Show that $\alpha^2 + \beta^2 = 5$. [2]

(iii) Hence find a quadratic equation which has roots $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$. [4]

8 **(i)** Show that $(r+2)! - (r+1)! = (r+1)^2 \times r!$. [3]

(ii) Hence find an expression, in terms of n, for

$$2^2 \times 1! + 3^2 \times 2! + 4^2 \times 3! + \ldots + (n+1)^2 \times n!.$$ [4]

(iii) State, giving a brief reason, whether the series

$$2^2 \times 1! + 3^2 \times 2! + 4^2 \times 3! + \ldots$$

converges. [1]

9 The matrix **C** is given by $\mathbf{C} = \begin{pmatrix} 0 & 3 \\ -1 & 0 \end{pmatrix}$.

(i) Draw a diagram showing the unit square and its image under the transformation represented by **C**. [2]

The transformation represented by **C** is equivalent to a rotation, R, followed by another transformation, S.

(ii) Describe fully the rotation R and write down the matrix that represents R. [3]

(iii) Describe fully the transformation S and write down the matrix that represents S. [4]

10 The matrix **D** is given by $\mathbf{D} = \begin{pmatrix} a & 2 & 0 \\ 3 & 1 & 2 \\ 0 & -1 & 1 \end{pmatrix}$, where $a \neq 2$.

(i) Find $\mathbf{D}^{-1}$. [7]

(ii) Hence, or otherwise, solve the equations

$$\begin{aligned} ax + 2y &= 3, \\ 3x + y + 2z &= 4, \\ -y + z &= 1. \end{aligned}$$ [4]

1.	(i) $a = -3$ (ii) $2a - 3 = 7$ or $3a - 6 = 9$ $a = 5$	B1 M1 A1	1 2 **3**	State correct value Sensible attempt at multiplication Obtain correct answer
2.	$x^2 - y^2 = 15$ and $xy = 4$ $\pm(4 + i)$	M1 A1 A1 M1 DM1 A1	 6 **6**	Attempt to equate real and imaginary parts of $(x + iy)^2$ and $15 + 8i$ Obtain each result Eliminate to obtain a quadratic in x^2 or y^2 Solve to obtain $x = (\pm)4$, or $y = (\pm)1$ Obtain only correct two answers as complex numbers
3.	$\frac{1}{4}n^2(n+1)^2 - \frac{1}{2}n(n+1)$ $\frac{1}{4}n(n-1)(n+1)(n+2)$	M1 M1 A1 M1 A1 A1	 6 **6**	Expand to obtain $r^3 - r$ Consider difference of two standard results Obtain correct unfactorised answer Attempt to factorise Obtain factor of $\frac{1}{4}n(n+1)$ Obtain correct answer
4.	(i) (ii)	B1 B1 B1 B1 B1 B1	 3 3	Circle Centre (1, -1) Passing through (0, 0) Sketch a concentric circle Inside (i) and touching axes Shade between the circles
5.	(i)	B1	1	Show given answer correctly

	(ii) $-1 \pm i\sqrt{3}$	M1 A1		Attempt to solve quadratic equation or substitute $x + iy$ and equate real and imaginary parts Obtain answers as complex numbers
	(iii)	A1 B1 B1 B1	3 3 **7**	Obtain correct answers, simplified Correct root on x axis, co-ords. shown Other roots in 2nd and 3rd quadrants Correct lengths and angles or co-ordinates or complex numbers shown
6.	(i) $u_{n+1} - u_n = 2n + 4$	B1 M1 A1	 3	Correct expression for u_{n+1} Attempt to expand and simplify Obtain given answer correctly
	(ii)	B1 M1 M1 A1 A1	 5 **8**	State $u_1 = 4$ (or $u_2 = 10$)and is divisible by 2 State induction hypothesis true for u_n Attempt to use result in (ii) Correct conclusion reached for u_{n+1} Clear,explicit statement of induction conclusion
7.	(i) $\alpha + \beta = -5 \quad \alpha\beta = 10$	B1 B1	2	State correct values
	(ii) $\alpha^2 + \beta^2 = 5$	M1 A1	 2	Use $(\alpha + \beta)^2 - 2\alpha\beta$ Obtain given answer correctly, using value of -5
	(iii) $x^2 - \frac{1}{2}x + 1 = 0$	B1 M1 A1 B1ft	 4 **8**	Product of roots = 1 Attempt to find sum of roots Obtain $\frac{5}{10}$ or equivalent Write down required quadratic equation, or any multiple.

8.	(i)	M1		Factor of $r!$ or (r + 1)! seen
		A1		Factor of $(r + 1)$ found
	$(r + 1)^2 r!$	A1	3	Obtain given answer correctly
	(ii)	M1		Express terms as differences using
		A1		(i)
		M1		At least 1st two and last term correct
	$(n + 2)! - 2!$	A1	4	Show that pairs of terms cancel
	(iii)	B1ft	1	Obtain correct answer in any form
			8	Convincing statement for non-converging, ft their (ii)
9.	(i) $\begin{pmatrix}0\\0\end{pmatrix}$ $\begin{pmatrix}0\\-1\end{pmatrix}$ $\begin{pmatrix}3\\0\end{pmatrix}$ $\begin{pmatrix}3\\-1\end{pmatrix}$	M1		For at least two correct images
		A1	2	For correct diagram, co-ords.clearly written down
	(ii) 90^0 clockwise, centre origin	B1 B1		Or equivalent correct description
	$\begin{pmatrix}0 & 1\\-1 & 0\end{pmatrix}$	B1	3	Correct matrix, not in trig form
	(iii) Stretch parallel to x-axis, s.f. 3	B1 B1		Or equivalent correct description, but must be a stretch for 2nd B1
	$\begin{pmatrix}3 & 0\\0 & 1\end{pmatrix}$	B1 B1	4	Each correct column
			9	

10.	(i)	M1		Show correct expansion process for
		M1		3 x 3
	$\Delta = \det \mathbf{D} = 3a - 6$	A1		Correct evaluation of any 2 x 2 det
		M1		Obtain correct answer
		A1		Show correct process for adjoint
		B1		entries
	$\mathbf{D}^{-1} = \frac{1}{\Delta}\begin{pmatrix} 3 & -2 & 4 \\ -3 & a & -2a \\ -3 & a & a-6 \end{pmatrix}$	A1	7	Obtain at least 4 correct entries in adjoint
	(ii) $\frac{1}{\Delta}\begin{pmatrix} 5 \\ 2a-9 \\ 5a-15 \end{pmatrix}$	M1		Divide by their determinant
		A1A1A1 ft all 3	4	Obtain completely correct answer
			11	
				Attempt product of form $\mathbf{D}^{-1}\mathbf{C}$, or eliminate to get 2 equations and solve
				Obtain correct answers, ft their inverse